If more generally

(34′) $$X = a_1 x_1 + a_2 x_2 + \dots a_m x_m = [a\,x],$$

where $x_1 = a_1 \pm r_1,\ x_2 = a_2 \pm r_2, \dots x_m = a_m \pm r_m,$

then we have by composition

(40′) $$\varphi(X) = \frac{\rho dX}{\sqrt{([a^2 r^2]\pi)}}\, e^{-\frac{\rho^2}{[a^2 r^2]}(X - [a\,a])^2},$$

(43′) $$X = [a\,a] \pm \sqrt{([a^2 r^2])}.$$

If

(34″) $$X = f(x_1, x_2, \dots x_m)$$

the above integration cannot be effected but an approximate solution can be given in that case which is the nearer perfect the smaller the probable errors $r_1, r_2, \dots r_m$. We have by Taylor's theorem, neglecting higher powers of increments $\triangle x_1, \triangle x_2, \dots \triangle x_m$

(34‴) $$X = f(a_1, a_2, \dots a_m) + f'(a_1)\triangle x_1 + f'(a_2)\triangle x_2 + \dots f'(a_m)\triangle x_m.$$

Within the range of $\triangle x_1, \triangle x_2, \dots \triangle x_m$, for which this form is exact enough, X is of the form (34′). In the integration however these increments have to pass from $+\infty$ to $-\infty$. If the probable errors are small this will make no sensible difference since the integral

$$\int_{\triangle}^{\infty} \frac{\rho d\triangle}{r\sqrt{\pi}}\, e^{-\frac{\rho^2}{r^2}\triangle^2}$$

approaches 0 the more rapidly the smaller r. This circumstance admits to a certain extent the treating of X as a linear function of $\triangle x_1, \triangle x_2, \dots \triangle x_m$ and we have

(40″) $$\varphi(X) = \frac{\rho dX}{\sqrt{([f'(a)^2 r^2]\pi)}}\, e^{-\frac{\rho^2}{[f'(a)^2 r^2]}[X - f'(a_1, a_2, \dots)]^2}$$

(43″) $$X = f(a_1, a_2, \dots a_m) \pm \sqrt{([f'(a)^2 r^2])}.$$

(To be continued.)

Note by S. W. Salmon.—In the note on Differential Calculus (p. 14), I wrote $\left(\frac{y - u'}{u - u'}\right)_{x=x'} = 1$. This needs to be proved. If the rate of motion of the point B is increasing, just before $x = x'$, $\left(\frac{y - u'}{u - u'}\right)$ is greater than 1, and just after $x = x'$, it is less than 1; therefore when $x = x'$, $\left(\frac{y - u'}{u - u'}\right) = 1$. If B's rate is decreasing, it may be proved in a similar manner that $\left(\frac{y - u'}{u - u'}\right)_{x=x'} = 1$.